A Parameterized Mesh Generation and Refinement Method for Finite Element Parameter Sweeping Analysis of Electromagnetic Devices

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Abstract — A parameterized mesh generation and refinement method is presented for parameter sweeping analysis of electromagnetic designs. The method is remeshing-free thus it can reduce the numerical simulation time significantly during finite element analysis (FEA) process. The idea of this parameterized mesh method is illustrated by a planar triangular element and high quality mesh can be obtained and kept by using this method. An example of mesh triangulation using the proposed method is given.

I. INTRODUCTION

Finite element method (FEM) is a numerical method based on polynomial interpolation in the Soblev space and mesh triangulation to generate the computational mesh where the unknowns are defined and the numerical solution is to be solved. FEM can handle all kinds of geometric shapes and can treat various boundary conditions conveniently and also there have been many computer aided engineering software packages which can significantly reduce the coding work during FEA. In the FEA process, the preprocessing is very important from which a suitable computational mesh for FEA is generated for further computation. For optimization problems in EM, usually the preprocessing programs will be called thousands of times to form mesh again and again which is a waste of time. In this paper we propose a parameterized mesh generation method which can generate a set of new mesh by simply reset all the coordinates of the original mesh according tothe parameters of the geometry. The previous solution can be interpolated easily by these parameters accordingly for further use.

II. ADAPTIVE PARAMETERIZED MESH GENERATION METHODS

Supposing there are geometry parameters p_1, p_2, …, p_N which will vary during parameter sweeping. We express them in a column matrix:

$$\{p\}=\begin{Bmatrix} p_1 \\ p_1 \\ \vdots \\ p_N \end{Bmatrix} \tag{1}$$

For simplicity, here 2-D FEM with triangular element is used as an example to show our basic idea. The coordinates of the vertex i in a finite element mesh can be expressed as linear functions of the parameters p_1, p_2, …, p_N:

$$x_i=[C_{ix0}\quad C_{ix1}\quad C_{ix2}\quad \cdots\quad C_{ixN}]\begin{Bmatrix} 1 \\ p_1 \\ p_2 \\ \vdots \\ p_N \end{Bmatrix}=[C_{ix}]\begin{Bmatrix} 1 \\ p \end{Bmatrix} \tag{2}$$

$$y_i=[C_{iy0}\quad C_{iy1}\quad C_{iy2}\quad \cdots\quad C_{iyN}]\begin{Bmatrix} 1 \\ p_1 \\ p_2 \\ \vdots \\ p_N \end{Bmatrix}=[C_{iy}]\begin{Bmatrix} 1 \\ p \end{Bmatrix} \tag{3}$$

where $[C_{ix}]$ and $[C_{iy}]$ are matrices with constant coefficients. For each vertex, not only the current coordinate values associated with current $\{p\}$ will be stored, these two matrixes will also be stored in the class of vertex. When $\{p\}$ changes, the coordinates of all vertices of the mesh will also be changed accordingly.

During mesh refinement, if a vertex k is added at the barycenter of the triangle element with three vertices i, j, m, its coordinates are:

$$\begin{aligned} x_k &= \left[\frac{C_{ix0}+C_{jx0}+C_{mx0}}{3}\quad \frac{C_{ix1}+C_{jx1}+C_{mx1}}{3}\quad \cdots\right. \\ &\qquad \left.\frac{C_{ixN}+C_{jxN}+C_{mxN}}{3}\right]\begin{Bmatrix} 1 \\ p \end{Bmatrix} \\ &= \frac{1}{3}[C_{ix}+C_{jx}+C_{mx}]\begin{Bmatrix} 1 \\ p \end{Bmatrix} \end{aligned} \tag{4}$$

$$\begin{aligned} y_k &= \left[\frac{C_{iy0}+C_{jy0}+C_{my0}}{3}\quad \frac{C_{iy1}+C_{jy1}+C_{my1}}{3}\quad \cdots\right. \\ &\qquad \left.\frac{C_{iyN}+C_{jyN}+C_{myN}}{3}\right]\begin{Bmatrix} 1 \\ p \end{Bmatrix} \\ &= \frac{1}{3}[C_{iy}+C_{jy}+C_{my}]\begin{Bmatrix} 1 \\ p \end{Bmatrix} \end{aligned} \tag{5}$$

That is to say, the expansion coefficients of (x_k,y_k) under $\{1, p_1, p_2, \ldots, p_N\}$ is $(\frac{1}{3}\sum_{l=i,j,m}[C_{lx}],\frac{1}{3}\sum_{l=i,j,m}[C_{ly}])$ if it is added at the barycenter of the triangle element. For general case, the newly added vertex's coefficient is computed by $(\sum_{l=i,j,m}\lambda_l[C_{lx}],\ \sum_{l=i,j,m}\lambda_l[C_{ly}])$, where the weights $\{\lambda_l\}$ are the area coordinates of the vertex $k(x_k,y_k)$ in the triangle with vertices i, j, m.

With the matrices $[C_x]$ and $[C_y]$ for each vertex, during optimal design process, all coordinates of the original vertices and the added vertices in the refined mesh will be changed automatically whenever the parameters $\{p\}$ vary. The shape of refined mesh may be changed but its topology will keep the same and its mesh quality will still be high. No mesh regeneration is required therefore the computing time for regeneration will be greatly reduced. The solutions on each vertex will be carried over from previous mesh to current mesh and no mapping of vertices is required.

In practice, after inputting the initial coarse mesh with $[C_x]$ and $[C_y]$ for each vertex, we will refine the initial coarse mesh successively to generate high-quality computational mesh for FEM analysis. The following steps are implemented in turns:

(i) Firstly a set of uniformly distributed nodes in the (x,y) plane will be added to the initial mesh and the swapping diagonal operations are done also by the Delaunay method [1]. This mesh adaption operation is done only once and the aim of it is to improve the aspect ratio of the initial mesh. Then the following steps are done repeatedly until the desired mesh is obtained.

(ii) Secondly the domain boundaries and material interfaces are refined as described in [2-3] where the diagonal swapping operation is impossible.

(iii) At the last step we will mark each element by a quality factor (QF). If QF is poor, we insert a node at the middle of the longest edge of that element. The quality factor is an indicator of the shape of the triangular element and they can be calculated as (6) or (7) and some sample triangles are labeled with their QF in Fig. 1 [4], where a, b and c are the length of three sides of the triangle considered respectively and Δ is its area:

$$Q_1 = 8(s-a)(s-b)(s-c)/abc \tag{6}$$

$$Q_2 = 4\sqrt{3}\Delta/(a^2+b^2+c^2) \tag{7}$$

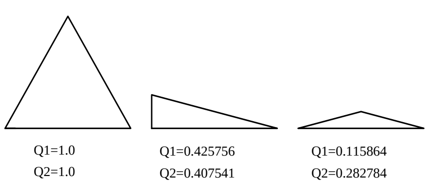

Fig. 1. Sample quality factor of triangles.

III. Examples and Results

In this paper, the proposed method is utilized to sweep the geometrical dimensions of an electromechanical levitation device, aiming to realize maximum magnetic force per unit volume of permanent magnet (PM). The initial mesh and material composition are shown in Fig. 2. After mesh refinement for three times, the mesh results are shown in Fig. 3. When the geometry dimensions change during sweeping process, the position of the nodes in the refined mesh will be varied accordingly, but its mesh quality are still kept high as shown in Fig. 4.

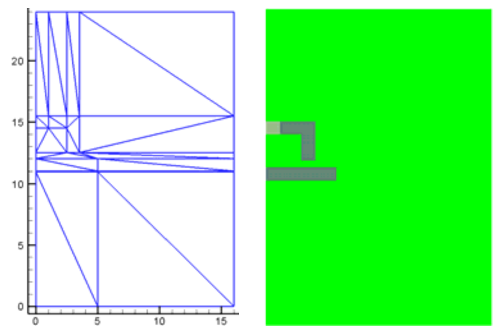

Fig. 2. The initial mesh and material composition.

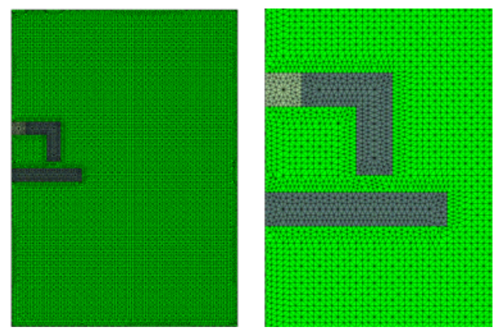

Fig. 3. The mesh after 3 refinements (14654 vertices, 28853 elements), the right is a closeup of the left.

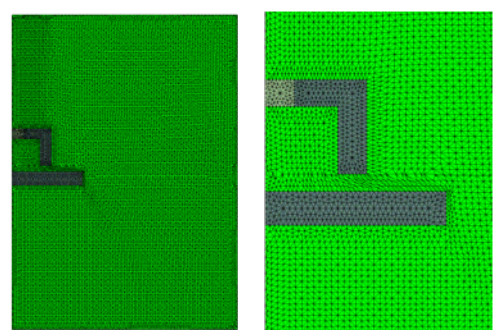

Fig. 4. Reset mesh after 3 refinements by changing parameters, the right is a closeup of the left.

IV. References

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